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The Higgs and top masses: why is the higgs mass $m_H^2 = m_Z m_t$?

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On the light of the recent LHC boson discovery, we present a simple computation of the ratio $\rho_t = m_Z m_t/m_H^2$. From the LHC combined m_H value, we get

$$\rho_t = 1.0022 \pm 0.007 \pm 0.009.$$

It is tempting to think that such a value, it is not a mere coincidence but, on naturalness grounds, a signal of some more deeper symmetry. In a model independent way, ρ_t can be viewed as the ratio of the highest massive representatives of the spin $(0, 1/2, 1)$ SM and, to a very good precision the LHC evidence tell us that $m_{s=1}m_{s=1/2}/m_{s=0}^2 \simeq 1$. Somehow the "lowest" scalar particle mass is the geometric mean of the highest spin 1, 1/2 masses. We review the theoretical situation of this ratio in the SM and beyond. In the SM such a ratio hints for a non-casual relation of the type $\lambda \sim g g_t$. Moreover, ρ_t together with $m_H \sim (m_W + m_t)/2$ could be interpreted as a hint for a role of the $SU(2)_c$ custodial symmetry in the explanation of the m_H/m_t ratio. Beyond the SM, Littlest Higgs Models, for example, where $\lambda \sim o(g^2, g_t^2)$ could accomodate such a relation.

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I. THE RATIO $\rho_t = m_Z m_t/m_H$

The problem of mass has two independent aspects in particle physics. The first, how mass arises, it is answered in the SM by the Higgs mechanism. The second aspect is why different elementary particles have their specific masses. Unless electromagnetic charge, there is no any, exact or approximate, known relation, structure or hierarchy among the masses of the SM elementary particles.

In the light of the recent results from the LHC coming from the experiments ATLAS and CMS, the parameter defined by the relation

$$\rho_t = \frac{m_Z m_t}{m_H^2} \tag{1}$$

where m_Z, m_t are the masses of the Z^0 gauge boson and the top quark and m_H is the Higgs mass has become experimentally measurable. Its current value can be estimated to be

$$\rho_t^{(exp)} = 1.0022 \pm 0.007 \pm 0.009 \tag{2}$$

where we have used the current values for [1]

$$m_Z = 91.1876 \pm 0.0021, \tag{3}$$
$$m_t = 173.5 \pm 0.6 \pm 0.8 \tag{4}$$

and the combined value of the boson masses presented by ATLAS and CMS [2, 3], $m_H = 125.6 \pm 0.4 \pm 0.5$ GeV/c^2. If the individual values for each ot the experiments are used instead, we get

$$\rho_t^{(exp)} = 1.0077 \pm 0.007 \pm 0.009 \qquad (m_{h,ATLAS}), \tag{5}$$
$$\rho_t^{(exp)} = 0.9965 \pm 0.007 \pm 0.007 \qquad (m_{h,CMS}) \tag{6}$$

for boson masses respectively $m_H = 125.3 \pm 0.4 \pm 0.5$ GeV/c^{2}and $m_H = 126.0 \pm 0.4 \pm 0.4$ GeV/c^2. The conclusion is that the experimental value of the ratio ρ_t is close to one with a precision or the order or less than 1%. This precision is not far from the precision at which the well known ratio $\rho = m_W^2/m_Z^2 \cos^2\theta_W$ is presentely measured, $\rho = 1.0008 \pm 0.001$ [1] with θ_W the Weinberg angle and m_W the charged electroweak gauge boson mass. The closeness of this parameter ρ_t to one might be merely a coincidence which will dissapear with any new measurement or might be not.

Note that the ratio would be exactly one for a boson mass (and nominal m_Z, m_t PDG masses) of

$$m_H(\rho_t = 1) \simeq 125.8 \quad \text{GeV/c}^2, \tag{7}$$

a value somewhere inbetween of the $125 - 126$ range of values currently measured by LHC and just on the borderline of the SM vacuum stability limits [11].

The ratio ρ_t would be still close to one, with a precision of 5%, if the Higgs mass appear finally anywhere in the range $m_H = 123 - 129$ GeV/c^2. If we vary the top mass in the range $m_t \sim 170 - 170$ GeV/c^{2}similar results are obtained.

Similar ratios involving other fermion masses instead of the top mass could be obviously defined, for example including all the fermions we could define ρ_Σ as

$$\rho_\Sigma = \frac{m_Z m_\Sigma}{m_H^2}, \tag{8}$$

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with

$$m_\Sigma^2 \;=\; \sum_f m_f^2 \qquad (9)$$

or including the third family quark doublet ($m_Q^2 = m_t^2 + m_b^2$) we could define the ratio

$$\rho_T \;\equiv\; \frac{m_Z m_Q}{m_H^2}, \qquad (10)$$

$$\;\simeq\; \rho_t \left(1 + 2\left(\frac{m_b}{m_t}\right)^2\right). \qquad (11)$$

In any case, any of these or similar ratios are deviated from ρ_t by a very moderate quantity $(m_b/m_t)^2 \simeq 10^{-3}$.

It is also interesting, or at least equally amusing, to consider an alternative way to express the closeness of the ratio ρ_t to one. If we consider the individual mass rations $m_Z/m_H, m_H/m_t$, their current experimental values are [1].

$$\frac{m_Z}{m_H} \;=\; 0.726 \pm 0.003, \qquad (12)$$

$$\frac{m_H}{m_t} \;=\; 0.724 \pm 0.005 \qquad (13)$$

where we have taken the LHC combined value of m_H. and PDG m_Z, m_t masses. Both ratios are the same at the level of 1% (and totally compatible at even higher precision according to present error bars). Very similar results are obtained if we use any of the ATLAS or CMS individual measurements

II. IN THE SM

The latest LHC measurements point to a preferred discovery of a neutral boson of spin not one, i.e. of spin 0 or 2. In a model independent way, thus the quantity ρ_t can be viewed as the ratio of the highest massive representatives of the spin $(0, 1/2, 1)$ particles of the Standard Model and, to a very good precision (assuming the new particle is a scalar for concreteness) the experimental evidence tell us that

$$\rho_t \sim \frac{m_{s=1} m_{s=1/2}}{m_{s=0}^2} \;\simeq\; 1. \qquad (14)$$

Somehow the mass of the "lowest" scalar particle mass is the geometric mean of the highest spin 1 and spin 1/2 masses.

Let us now assume that the new particle is a scalar Higgs boson. In the Standard Model (SM) with a Higgs sector consistent of one Higgs doublet Φ and scalar potential

$$V_{SM} \;=\; \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2, \qquad (15)$$

all masses are induced by spontaneous symmetry breaking and are proportional to the Higgs vaccuum expectation $< \phi_0 >= v/\sqrt{2}$ where

$$v^2 \;=\; -\frac{\mu^2}{\lambda} = \frac{1}{\sqrt{2} G_F} = (246.218)^2 \quad \left(\mathrm{GeV/c^2}\right)^2 \qquad (16)$$

The tree level top, gauge and Higgs boson masses are given in terms of v and their respective Yukawa couplings

$$m_W \;=\; g\frac{v}{2}, \quad m_Z = \sqrt{g^2 + g'^2}\frac{v}{2} \qquad (17)$$

$$m_t \;=\; g_t \frac{v}{2}, \qquad (18)$$

$$m_H^2 \;=\; -2\mu^2 = 2\lambda v^2. \qquad (19)$$

Moreover we have $g' = g \tan\theta_W$ or $\sqrt{g^2 + g'^2} = g/\cos\theta_W, G_F m_W^2/\sqrt{2} = g^2/8$.

In terms of these quantities the tree level mass ratio ρ_t is simply given by

$$\rho_t^{SM} \;=\; \frac{m_Z m_t}{m_H^2} = \sqrt{g^2 + g'^2}\frac{v^2 g_t}{4\sqrt{2} v^2 \lambda} \qquad (20)$$

$$\;=\; \frac{1}{4\sqrt{2}} \frac{\sqrt{g^2 + g'^2} g_t}{\lambda} \qquad (21)$$

$$\;=\; \frac{1}{4\sqrt{2}} \frac{g g_t}{\cos\theta_W \lambda}. \qquad (22)$$

Numerically, the top yukawa, quartic coupling and other related ratios at tree level approximation are experimentally measured to be (1σ errors):

$$g_t^{(exp)} \;=\; 1.409 \pm 0.007, \qquad (23)$$

$$\lambda^{(exp)} \;=\; 0.130 \pm 0.001, \qquad (24)$$

$$\left(\frac{\lambda}{g_t^2}\right)^{(exp)} \;=\; \frac{1}{8}\left(\frac{m_H}{m_t}\right)^2 = (6.6 \pm 0.1) \times 10^{-2}, \qquad (25)$$

$$\left(\frac{g^2 + g'^2}{\lambda}\right)^{(exp)} \;=\; 8\left(\frac{m_Z}{m_H}\right)^2 = 4.21 \pm 0.03. \qquad (26)$$

In the SM, the Higgs selfcoupling λ is undetermined. However, assuming that the value $\rho_t^{(exp)} \simeq 1$ is not merely a coincidence, the relation Eq.(21) tell us that the scalar self-coupling and the spin 1 and spin 1/2 top couplings are subject to the tree level equality

$$\lambda \;\simeq\; c\sqrt{g^2 + g'^2} g_t \qquad (27)$$

$$\;\simeq\; c g g_t, \qquad (28)$$

where c is a numeric factor of order $\sim o(1)$.

The tree level relation (21) is affected by SM quantum corrections. Including one loop corrections, the three

[1] "God" or "golden" particle?. The difference between any of the m_H/m_Z, m_Z/m_H ratios and the Golden Ratio $(\sqrt{5} + 1)/2$ is a "mere" 15%. Equality would be exact if $m_t = m_H + m_Z$

level relations above should be replaced, in particular by (where μ_0 the renormalization scale, $\mu_0 \sim m_Z - m_t$)

$$g_t(\mu_0) = \frac{\sqrt{2}m_t}{v}\left(1+\delta_t(\mu_0)\right), \quad (29)$$

$$\lambda(\mu_0) = \frac{\sqrt{m_H^2}}{2v^2}\left(1+\delta_\lambda(\mu_0)\right), \quad (30)$$

we consider negligible the running of the gauging couplings $g_i(\mu_0)$. The first order corrected ratio is then, using expresions (29,30),

$$\rho_t^{SM} = \frac{m_Z m_t}{m_H^2} \quad (31)$$

$$= \frac{1}{4\sqrt{2}}\frac{g g_t}{\cos\theta_W \lambda}\frac{1+\delta_\lambda}{1+\delta_t} \quad (32)$$

$$\simeq \rho_t^0\left(1+\delta_\lambda-\delta_t\right). \quad (33)$$

The top yukawa δ_t can be written as $\delta_t = \delta_t^{QCD} + \delta_t^w$. The corrections are ([4] and references therein), ignoring logarithm terms,

$$\delta_\lambda = \frac{1}{16\pi^2}c_\lambda \lambda, \quad (34)$$

$$\delta_t^w = \frac{1}{16\pi^2}\frac{c_t}{8}g_t^2, \quad (35)$$

$$\delta_t^{QCD} = (-1/(3\pi^2))g_s^2, \quad (36)$$

with the numerical coefficients $c_\lambda \simeq 25/2 - 9\pi/(2\sqrt{3}) \simeq 4.3$, $c_t \simeq 6.1$. Thus

$$\frac{\delta_\lambda}{\delta_t^w} \simeq \frac{c_\lambda}{c_t}\left(\frac{m_H}{m_t}\right)^2 \simeq 0.3. \quad (37)$$

Then

$$\rho_t = \rho_t^0\left(1+c_1\lambda - c_2 g_t^2 - c_s g_s^2\right). \quad (38)$$

The correction $\delta_t^{QCD} \sim 5\%$ is the most important one, acting to diminish slightly the ratio. Both corrections, $\delta_t^w, \delta_\lambda$, are of opposite sign and very small, of the order of 1%.

1. SM Renormalization group equations.

We explore here the behaviour of the mass ratio (2,21) at higher scales. We consider first a reduced system of one-loop renormalization group equations where only the Higgs self-coupling λ and the top Yukawa coupling g_t appear. All the other couplings are considered very small or not running at all. The RGE equations for the individual couplings take the form (see for example [5–8]) (with $t = \log(\mu/\Lambda)$, expression valid for high, but no so high, scales $\mu >> m_t, m_H$, or for $\Lambda \to \infty$):

$$\frac{dg_t^2}{dt} = \frac{9}{16\pi^2}g_t^4, \quad (39)$$

$$\frac{d\lambda}{dt} = \frac{6}{16\pi^2}\left(4\lambda^2 + 2\lambda g_t^2 - g_t^4\right). \quad (40)$$

If we introduce the variable

$$R = \frac{\lambda}{g_t^2}, \quad (41)$$

the RGE equations for g_t, R and $\rho_t(t)$ become decoupled with nested solutions, $g_t = g_t(\mu)$,$R = R(g_t)$,$\rho_t = \rho_t(R)$. In addition to Eq.(39), we have

$$g_t^2\frac{dR}{dg_t^2} = \frac{1}{3}f(R), \quad (42)$$

$$\frac{d\rho_t}{dR} = -\frac{3\rho_t}{2f(R)}\left(1+\frac{2f(R)}{3R}\right). \quad (43)$$

with $f(R) = 8R^2 + R - 2$. The equations (39,42,43) can be solved explicitly, in particular for the ratio ρ_t we can write

$$\rho_t = k\left(\frac{R_0 - R}{R_1 + R}\right)^{R_0 - R_1} R^2,$$

where R_0, R_1 are the fixed points of the equation (42), $f(R_{0,1}) = 0$. For a light Higgs and large top mass the ratio R is small, at low scales $R^{exp} \sim 10^{-1}$, Eq.(25). For such a small R the solution of the differential equations is approximately:

$$R(g_t) = R_c - \frac{4}{3}\log g_t, \quad (44)$$

and

$$\rho_t \sim kR^2 \sim (R_c - \frac{4}{3}\log g_t)^2 \sim kR_c^2 \sim \rho_t^0. \quad (45)$$

At large energies ($\mu >> m_t$, as long as $R > 0$ or $\lambda > 0$), the ratio $\rho_t(\mu)$ keeps approximately constant, only sligtly decreasing with the logarithm of g_t.

If we consider a reduced Higgs-top-strong system where the λ, g_t, g_s are non-vanishing and allowed to run together with the ratios R, ρ_t. One ends with a similar system of equations where the evolution of ρ_t is of the type $g_t^2 d\rho_t/g_t^2 \sim \rho_t h(R, g_t^2)$ and similar results are obtained.

At higher energies, and for more quantitative results, a full treatment is needed. Present state-of-the-art NLO and NNLo constraints on SM vacuum stability [11] seems to guarantee stability, and then a reasonably stable, positive, value for the quartic coupling, for a Higgs mass $m_H \sim 126$ GeV/c^{2}and to very high scales.. If we assume a stable behaviour for λ and ignoring the very modest variation of the coupling factor $g^2 + g'^2$,

$$\rho_t(\mu) \sim \rho_t^0\frac{g_t(\mu)}{g_t^0}.$$

the variation of the mass ratio ρ_t is governed by the top Yukawa up to scales where new physics is expected to emerge.

III. BEYOND THE SM

We expect new physics that cuts off the divergent top, gauge and higgs loop contributions to the Higgs Mass at scales $\lesssim$ 10 TeV. Many different possibilities have been well explored, they usually include, more or less ad-hoc, new particles with properties tightly associated to those of the SM. Some of these possibilities are for example (and any combinations among them)[9, 10]: a) The new particles are just the, softly broken, SUSY, superpartners with couplings and Yukawas strongly dictated by supersymmetry and the soft breaking itself. b) The Higgs is a composite resonance, or c) The "Little" Higgs is a pseudo-Nambu-Goldstone boson with respect a "softly" broken approximate global symmetry. This scalar sector is accompanied by some new particles belonging to enlarged multiplets together with the SM particles.

It is a general feature that, in all or most of these models, the quartic self coupling, and then the Higgs mass, is related to the gauge coupling constants and to the top yukawa in a more or less explicit way, reminding of the relation (25) suggested by the experimental evidence $\rho_t \simeq 1$. The reason is clear [9], the new one-loop which are proportional to the couplings of the SM gauge sector (or to a subsector of an enlarged gauge sector) have to match and cancel the top and the other cuadratic loops.

We will briefly review the situation in the MSSM and Littlest Higgs scenarios. In the MSSM, the tree level top, gauge and lowest Higgs boson masses together their respective Yukawa couplings are given by the expressions

$$v^2 = v_1^2 + v_2^2, \quad \tan\beta = v_2/v_1, \tag{46}$$

$$m_W = g\frac{v}{2}, \quad m_Z = \sqrt{g^2+g'^2}\frac{v}{2} \tag{47}$$

$$m_t = g_t\frac{v}{2}\sin\beta, \tag{48}$$

$$m^2_{H\,tree} = -2\mu^2 = 2\lambda v^2. \tag{49}$$

where the tree level Higgs quartic coupling is determined in terms of the gauge couplings

$$\lambda = (g^2+g'^2)\cos^2 2\beta. \tag{50}$$

Quantum corrections to the Higgs quartic coupling are very important. They lead for an expression for the lower neutral Higgs mass, of the form [12]

$$\begin{aligned} m_H^2 &= m_Z^2\cos^2 2\beta + \delta m_H^2 && (51)\\ &= m_Z^2\cos^2 2\beta + f\frac{3G_F}{\sqrt{2}\pi^2}m_t^4 && (52)\end{aligned}$$

where the factor f include logarithmic corrections, mainly associated to stops. From the expression (52) and from $m_H^2 = 2\lambda(\mu)v^2$ we can extract an improved value for the quartic effective coupling

$$\lambda(\mu) = \frac{m_H^2}{2v^2}\left(1+\delta_\lambda(\mu)\right). \tag{53}$$

The effective quartic coupling is of the natural size $\lambda \sim o(g^2, g_t^4)$. In terms of these quantities the mass ratio ρ_t is simply given by

$$\begin{aligned} \rho_t^{MSSM} &= \frac{m_Z m_t}{m_H^2} = && (54)\\ &= \frac{\sqrt{g^2+g'^2}g_t\sin\beta}{(g^2+g'^2)\cos^2 2\beta + g_t^4\sin^4\beta 3f/\pi^2}. && (55)\end{aligned}$$

In the context of the MSSM, the experimental evidence $\rho_t \simeq 1$ is a strong hint for the existence of a relation among the parameters of the expression above, principally top Yukawa and $\tan\beta$ together with the gauge couplings.

As a second illustrative example, let us mention the "Littlest" Higgs scenario [9]. Here the usual Higgs doublet, is the lightest of a set of pseudo goldstone bosons in an non-linear sigma model including in its gauge group different $SU(2)\times U(1)$ factors. The product group is broken to the diagonal, identified as the SM electroweak gauge group. The top Yukawa coupling generates a negative mass squared triggering electrowak symmetry breaking. New particles are added, in particular heavy top partners, which cancel the one loop quadratically divergent corrections. The quartic self coupling is related to the coupling constants of the gauge sector and to the top Yukawa with a natural size

$$\lambda \sim o(g^2, g_t^2),$$

reminding the expression suggested by experimental evidence $\lambda \sim g g_t$. Particular scenarios can be tuned so that either the gauge contributions or the top Yukawas dominate the Higgs quartic and $m_H \sim m_Z$ or $m_H \sim m_t$ as extreme cases. In fact we have seen, according to Eq.(2), that nature chooses, to a very high precision, just the geometric average of both extreme cases $m_H = \sqrt{m_Z m_t}$. It seems plausible that a Little Higgs scenario can be found where this value appears naturally.

IV. SUMMARY AND CONCLUSIONS

In this short note we have presented some simple computations associated to the ratio of the product of Z^0 and top masses to the squared Higgs mass, ρ_t. We have shown how this ratio is suprisingly and robustly close to the unity at the 10^{-3} level. The Higgs mass seems to be just the geometrical mean of the m_Z and m_t masses.

In a model independent way, ρ_t can be viewed as the ratio of the highest massive representatives of the spin $(0, 1/2, 1)$ particles of the Standard Model and, to a very good precision (assuming the new particle is a scalar) the experimental evidence tell us that

$$\frac{m_{s=1}m_{s=1/2}}{m^2_{s=0}} \simeq 1. \tag{56}$$

Somehow the mass of the "lowest" scalar particle mass is the geometric mean of the highest spin 1 and spin 1/2 masses.

We have briefly reviewed the theoretical predictions of this ratio in the SM and beyond. In the SM, the Higgs selfcoupling λ is undetermined. However, assuming that the value $\rho_t^{(exp)} \simeq 1$ is not merely a coincidence, the relation Eq.(21) tell us that the scalar self-coupling and the spin 1 and spin 1/2 top couplings are subject to the tree level equality

$$\lambda \; \simeq \; c\sqrt{g^2+g'^2}g_t \simeq cgg_t, \qquad (57)$$

where c is a numeric factor of order $\sim o(1)$. Such a relation is not very much affected by quantum effects at least up to scales $\mu \sim m_t$ or slightly higher.

Moreover, the value $\rho_t \simeq 1$ together with the also experimental relation $m_H \sim (m_W + m_t)/2$ could be interpreted as a hint for an instrumental role of the $SU(2)_c$ custodial symmetry in the explanation of the m_H/m_t ratio [13, 14].

In all or most of the models beyond the SM where new particles are introduced to cut off the quadratically divergent loops, the quartic self coupling is related to the gauge coupling constants and to the top yukawa in a more or less explicit way, reminding of the relation $\lambda \sim gg_t$ suggested by the experimental evidence $\rho_t \simeq 1$. The reason for this is that the new one-loop which are proportional to the couplings of the SM gauge sector have to match and cancel the top and the other cuadratic loops. In "Little" Higgs models, for example, the quartic self coupling is related to the coupling constants of the gauge sector and to the top Yukawa with a natural size

$$\lambda \sim o(g^2, g_t^2),$$

Particular scenarios can be tuned so that either the gauge contributions or the top Yukawas dominate the Higgs quartic and $m_H \sim m_Z$ or $m_H \sim m_t$ as extreme cases. In fact we have seen that nature seems to choose, to a very high precision, just the geometric average of both extreme cases $m_H = \sqrt{m_Z m_t}$. It seems plausible that a Little Higgs scenario can be found where this value appears naturally. Approximate accidental global symmetries related to the Little Higgs scenario could play a role in the understanding of the ρ_t ratio, as the global custodial $SU(2)_c$ symmetry [13] plays for the ρ ratio.

The closeness of the parameter ρ_t to one might be merely a coincidence or an artifact of the early status of the Higgs discovery, which will dissapear with any new measurement or might be not. It is tempting to think that such a value of ρ_t is, on naturalness grounds, a signal of some more deeper symmetry.

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